Hadron spectroscopy from lattice QCD.

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Overview of the talk

I think that hadron spectroscopy is the study of bound states of quarks and gluons. I will review lattice QCD calculations of novel states such as glueballs and hybrid mesons.
Plan of the talk:

- Why look for glueballs and hybrid mesons
- Brief introduction to lattice QCD.
- Lattice QCD and nuclear physics
- Masses and widths of exotic mesons.
- Open decay channels.
- 0^{++} glueballs and the σ

The talk will be largely be based on the papers below:

- Decay width of light quark hybrid meson from the lattice. (C. McNeile and C. Michael, Phys.Rev.D73:074506,2006. hep-lat/0603007
- Hard hadron spectroscopy, C. McNeile, invited talk at lattice 2007 conference (arXiv:0710.0985).

Going beyond the quark model.

The quark model (as described in the review section of the PDG) claims that light quark spectroscopy is “explained” by

- $\overline{\psi}\Gamma\psi$ Mesons
- $\psi\psi\psi$ Baryons

In QCD there are additional possible colour singlet operators that may form bound states:

- Glueballs: lumps of glue.
- Hybrids mesons: quark and anti-quark with excited glue.
- Multiquark states.

Is there any evidence for the above states in nature? Can theory accurately compute non-perturbative results.

Some background to lattice QCD

To solve QCD for bound state properties “all” that is required is to compute

$$c_{ij}(t) = \frac{1}{Z} \int du \int d\psi \overline{d\psi} O(t)_i O(0)_j^\dagger e^{-S_F - S_G}$$

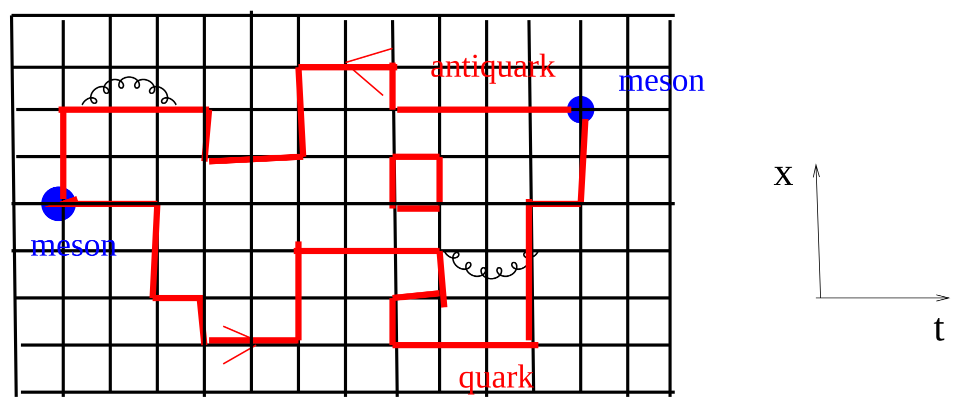
- The path integral is regulated by the introduction of a space-time lattice. Lattice calculations are now being done at several lattice spacings and volumes.
- The multi-dimensional integral is computed in Euclidean space using Monte Carlo techniques on the computer (similar $\int_{-\infty}^{-\infty} x^2 e^{-mx^2 - \lambda x^4} dx$).
- The dynamics of the gluon and quarks is in the Dirac action (S_F) and gauge action (S_G).

Pictures of lattice QCD (part of MC)

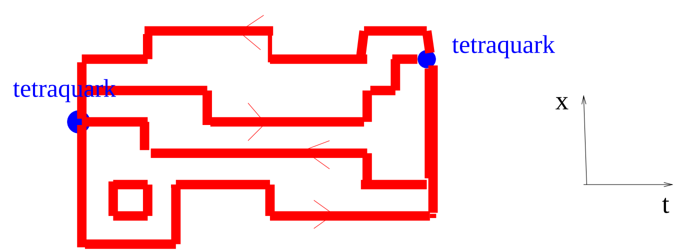
There is a physical picture (part of the Monte Carlo) of the process. For the meson interpolating operator

$$O(x,t) = \overline{\psi_1}(x,t)\psi_2(x,t)$$

where 1 and 2 are different flavours.



For tetraquark interpolating operator such as $\overline{q}\gamma_5 q \overline{q}\gamma_5 q$



QCD and nuclear physics

What stops us studying nuclei directly on the lattice?
In nucl-th/0611038 Martin Savage estimates the cost of doing the Wick contractions for a Nuclei and atomic mass A and charge Z as

$$\text{Cost} \sim (A+Z)!(2A-Z)!$$

The lattice volume needs to increase as well (Urbach, arXiv:0710.1517).

$$\text{Cost} \sim \pounds 6,000,000 \left(\frac{\text{20 MeV}}{m_q}\right)^1 \left(\frac{L}{3\ fm}\right)^5 \left(\frac{0.1\ fm}{a}\right)^6$$

Normally electromagnetic effects are not included, partly because EM effects in hadron masses are at the 5 MeV level.
Savage suggests a program of matching lattice QCD to various effective field theories and then using that as input to GFMC (Green's Function Monte Carlo), Coupled Cluster calculations.

A consumers view of lattice QCD

I will present results from older lattice QCD calculations that used quenched QCD (no dynamics of sea quarks) or the dynamical calculations had 500 MeV pions.

Now we are solving full QCD, but need to control the parameters. For example, the European Twisted Mass Collaboration (ETMC) have reached the parameters below

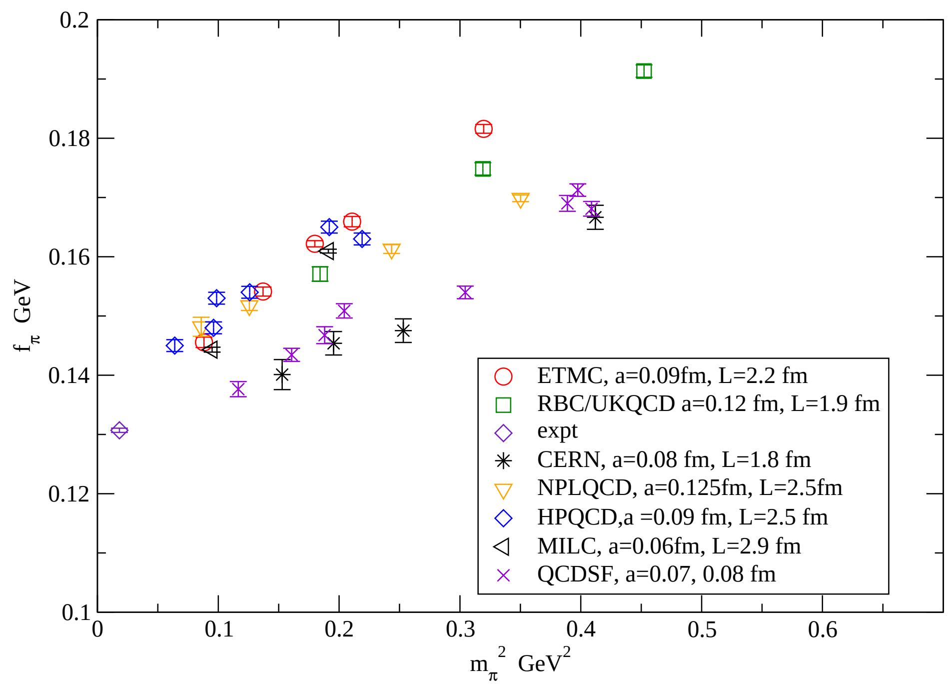
- Two flavours of sea quarks with range of pion masses 270 to 520 MeV. (2+1+1 flavours in this next year).
- Statistical errors under 1 % level for pseudo-scalar mesons.
- Two lattice spacings 0.07 and 0.09 fm.
- Finite volume effects are an issue. Volumes between 2.2 and 2.9 fm.

Other collaborations have reached similar parameters. The newer results are for simpler systems: pseudo-scalar mesons, kaons..

Validation, can you trust us?

Summary of the pion decay constant (f_π) as a function of square of the pion mass.

$$\Gamma(\pi^- \rightarrow e^- + \overline{\nu}_e) \propto \frac{G_F^2 \mid V_{ud} \mid^2 f_\pi^2}{m_\pi^3} m_l^2 (m_\pi^2 - m_l^2)^2$$



Exotic mesons

States with exotic quantum numbers J^{PC}= 1^{-+}, 0^{+-}, 2^{+-}, 0^{--} are definitely not described by the quark model (that predicts $C = (-1)^{l+s}$, $P = (-1)^{l+1}$).

- Hybrid exotic mesons with $\overline{q}q$ plus excited glue.
- Multiquark operators ($\overline{qq}qq$).

Interpolating operators used in lattice QCD calculations of hybrid meson spectroscopy.

1^{--}	$\overline{\psi}\gamma_j\psi$
1^{--}	$\epsilon_{ijk}\overline{\psi}\gamma_5 F_{ij}\psi$
1^{-+}	$\overline{\psi}\gamma_j F_{jk}\psi$
1^{-+}	$\overline{\psi}\gamma_j\gamma_5\psi$ $\overline{\psi}\gamma_5\psi$

where F is the QCD field strength tensor.

Experimental results

From the review by Klempt, hep-ph/0404270.

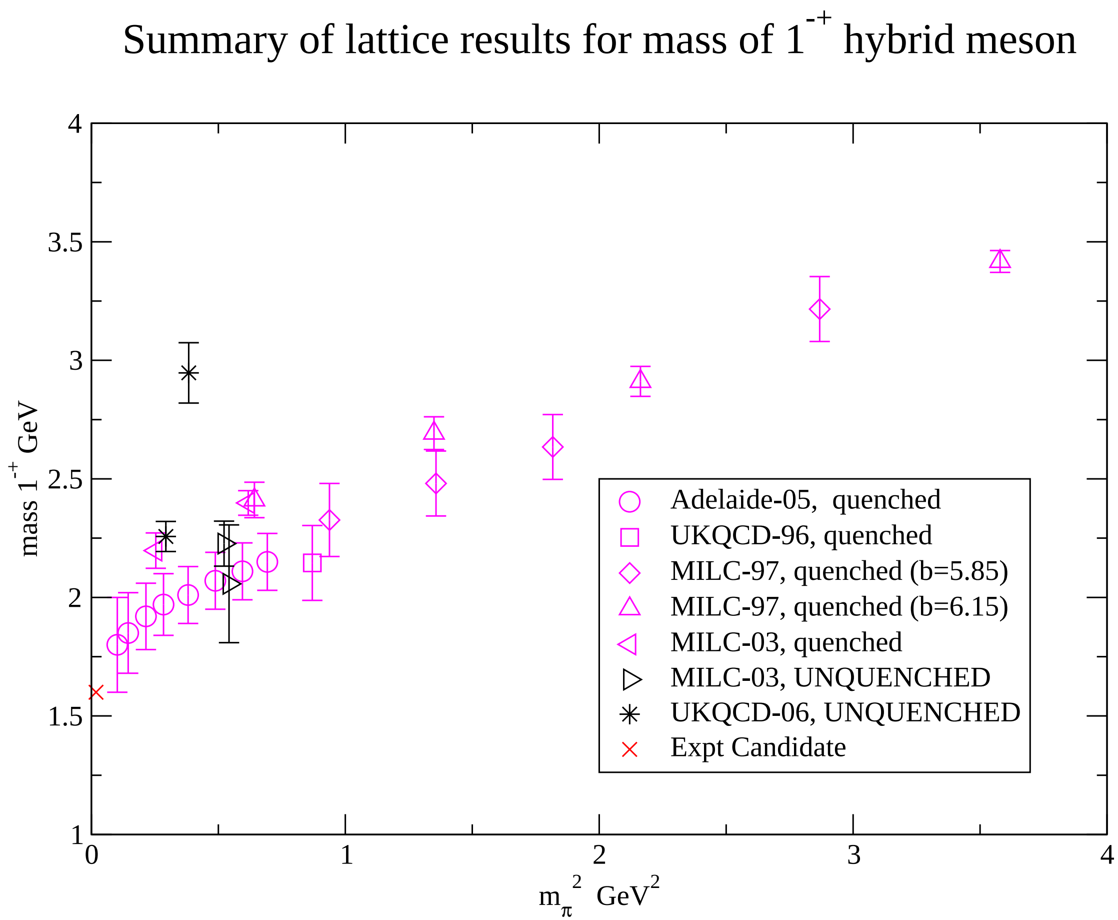
Experiment	Mass MeV	decay	width MeV
BNL,CBar	~ 1400	$\eta\pi$	~ 300
CBar	~ 1440	$\rho\pi$	~ 400
BNL,CBar	~ 1600	$\rho\pi$ + ...	~ 170
BNL	~ 2000	$f_1\pi$ + ...	~ 300

Table 1: Overview of experimental results for 1^{-+}.

The experimental analysis of the data can be controversial. Not everyone believes the state at 1400 Mev, but it is still mentioned in the "Non-$q\overline{q}$ mesons" review section of the PDG. (There is a state called Y(4260) in charmonium that some people think is a 1^{--} hybrid).

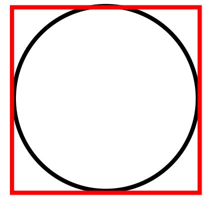
1^{-+} masses.

Lattice mostly still predicts lightest 1^{-+} state to be at 1.9(2) GeV.



Is the lattice 1^{-+} a non-exotic 4^{-+} ?

Issue recently raised by Dudek et al. (arXiv:0707.4162 [hep-lat])



Symmetries of lattice are subgroup of continumm group, so care is required in construction of operators with J^{PC}.

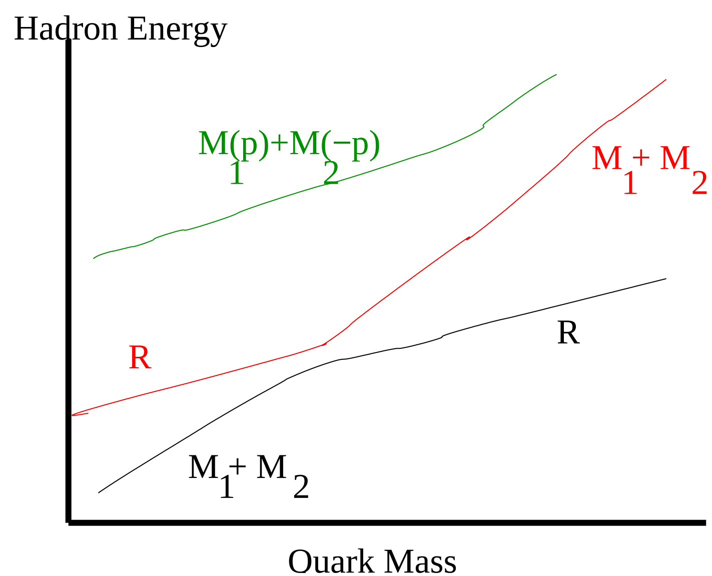
J	Lattice Rep
0	A_1
1	T_1
2	$E\ T_2$
3	$A_2\ T_1\ T_2$
4	$A_1\ E\ T_1\ T_2$

The $\overline{\psi}\gamma_i\psi$ operator lives in T_1, so can couple to 1^{--} and 3^{--} (UKQCD hep-lat/9605025). Current results for the masses of 1^{-+} states are "lower bounds", but doesn't improve agreement between lattice and expt at 1.6 GeV. Maybe important for charm 1^{-+} ?

Cartoon of meson decay

Resonance decay $R \to M_1 + M_2$

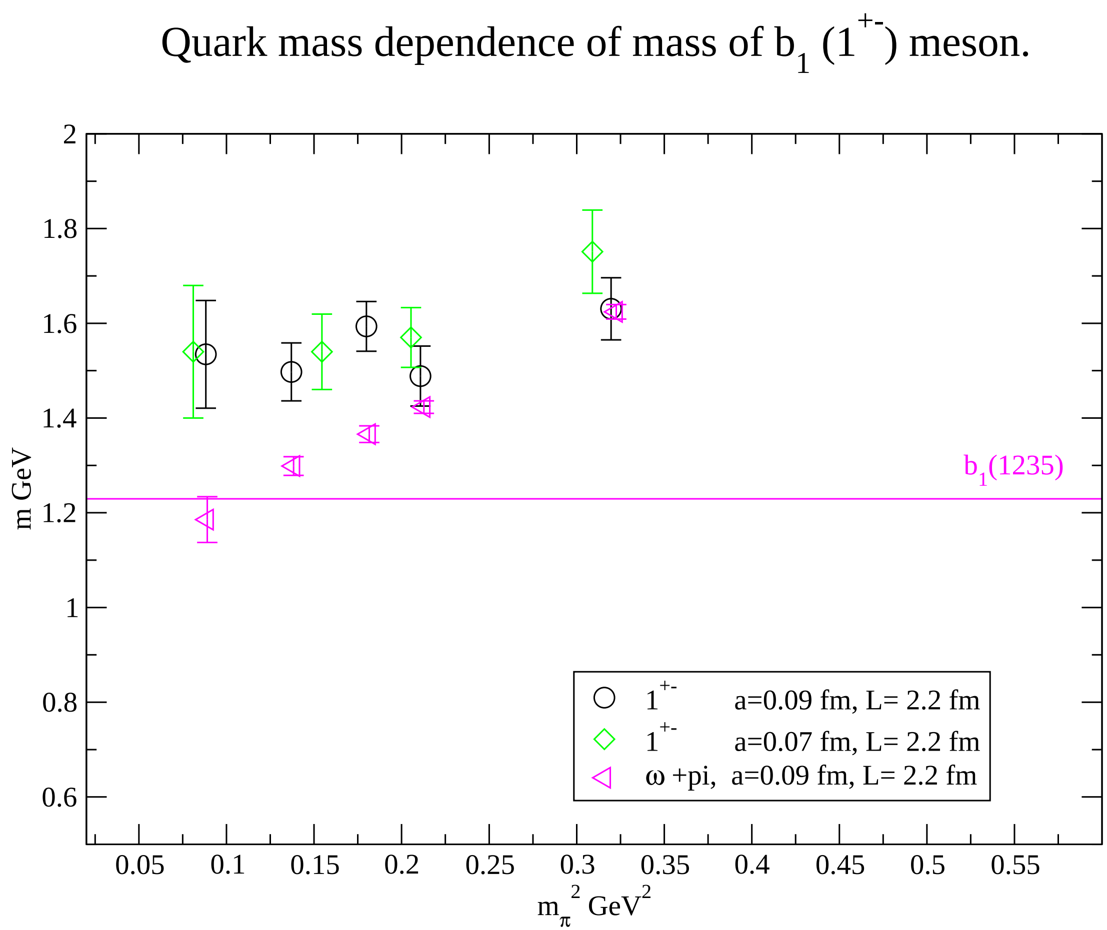
- S-wave decay, decay threshold $M_R = M_1 + M_2$
- P-wave decay, decay threshold $M_R = \sqrt{M_1^2 + p^2} + \sqrt{M_2^2 + p^2}$, p = $\frac{2\pi}{L}$ (~ 500 MeV)



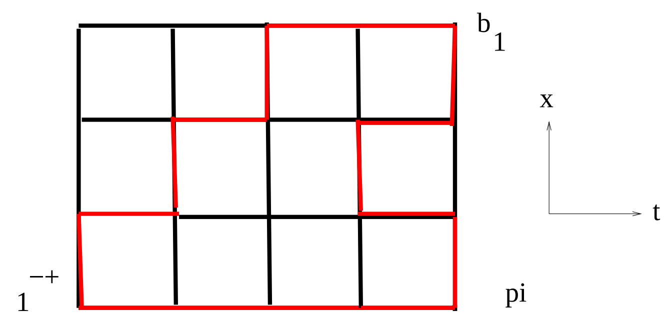
(Elegant method proposed by Lüscher, Nucl. Phys. B364, 237, 1991. First application for ρ decay , ariv:0708.3705, by CP-PACS.) MILC (hep-lat/0104002) originally claimed evidence for a_0 decay to $\pi\eta$ from LGT.

Preliminary results from ETMC

The b_1 meson has the dominant decay $\omega\pi$, that we should see on the lattice. (The difference between the ρ and ω is disconnected and small.) This is an updated analysis to my lattice 2007 presentation.



$1^{-+} \to \pi b_1$ (hep-lat/0603007)



$$H(t) = \frac{(1^{-+} \mid b_1 \pi)}{\sqrt{(1^{-+} \mid 1^{-+})(b_1 \mid b_1)(\pi \mid \pi)}} \sim xt$$

With transition matrix element xa plug into Fermi Golden rule to get width Γ.

$$\Gamma / k = \frac{1}{\pi} (xa)^2 (L/a)^3 \frac{E(b_1) a E(\pi)}{E(b_1) + E(\pi)}$$

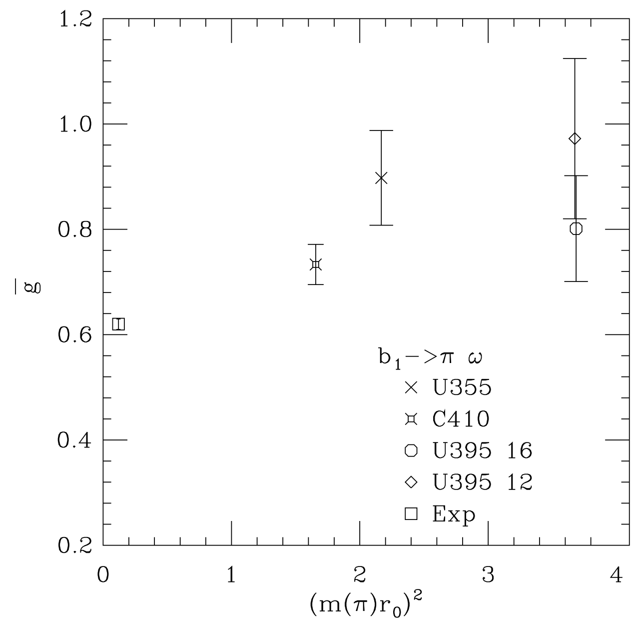
Decay widths from the lattice

Masses and decay widths are extracted at the same time from experimental data

Group	Decay	Result (MeV)	Expt (MeV)
UKQCD, hep-lat/0201006	$1_H^{-+} \to \chi_b + 0^{++}$	61(14)	?
UKQCD, hep-lat/0201006	$1_H^{-+} \to \chi_b + \eta_2$	< 1	?
CMN & Michael, hep-lat/0212020	$\rho \to \pi\pi$	153(70)	150
ETMC (preliminary)	$\rho \to \pi\pi$	~ 114	150
CP-PACS (arXiv:0708.3705)	$\rho \to \pi\pi$	140(27)	150
CMN & Michael, hep-lat/0603007	$1^{-+} \to \pi b_1$	400(120)	?
CMN & Michael, hep-lat/0603007	$1^{-+} \to \pi f_1$	90(60)	?
CMN & Michael, hep-lat/0201006	$b_1 \to \omega\pi$	230(60)	142(9)
Cook and Fiebig, hep-lat/0606005	$1^{-+} \to \pi a_1$	~ 60	?

Some systematics

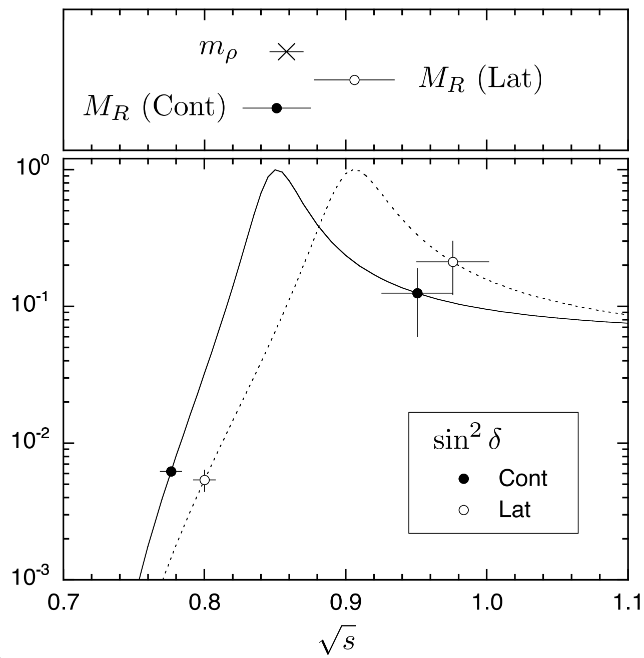
From McNeile and Michael (hep-lat/0603007) For decay $b_1 \rightarrow \omega\pi$ $\bar{g}^2 = \Gamma/k$



- Statistical errors large
- Large extrapolation

Phase shifts

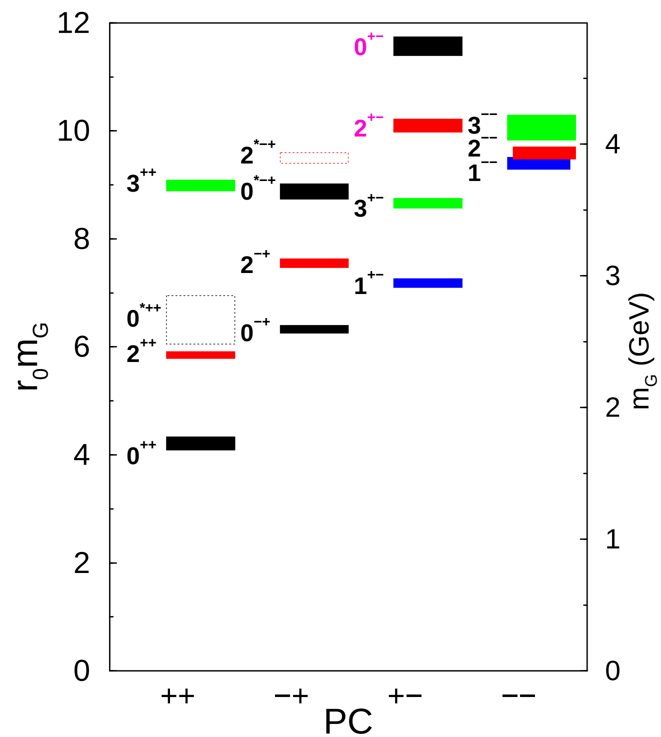
CP-PACS ((arXiv:0708.3705 [hep-lat]).) applied Lüscher's (Nucl. Phys. B364, 237, 1991) method to ρ meson to compute the scattering phase shift Extract width from fit to Breit-Wigner function. Later directly compare against experiment.



Cook and Fiebig, hep-lat/0606005 used the same formalism to attempt to compute scattering phase shifte for 1^{-+} meson.

In the beginning there were glueballs

In pure gauge theory there is rich spectrum of glueball states. From Morningstar and Peardon.



Is there are any evidence for the above states in nature?? Can we turn a theoretical triumph into a phenomenological triumph. I will focus on the 0^{++} state. (Only seen signal for 0^{++} and 2^{++} states from unquenched calculations (Hart and Teper)).

Singlet 0^{++} mesons and experiment

- $f_0(600)$ (σ) huge width, proposed 1955, entered PDG summary tables in 2002. M = 441^{+16}_{-8} MeV and Γ_σ = 544^{+18}_{-25} MeV from hep-ph/0512364, Caprini et al. (PDG summary has mass 400 to 1200 GeV).
- $f_0(980)$ maybe molecule ???
- $f_0(1370)$ hard to extract from data. M $\sim$ 1200 to 1500 MeV. Likes to decay to $\pi\pi$.
- $f_0(1500)$ Well established, likes to decay to $\pi\pi$.
- $f_0(1710)$ Likes to decay to $\overline{K}K$.

The quark model predicts that there should only be two states between 1300 and 1800 MeV.

Morningstar and Peardon obtained 1730(50)(80) MeV for the mass of the 0^{++} glueball.

Update of the Morningstar and Peardon results at finer lattice spacings by Chen et al.

hep-lat/0510074, $M_{0^{++}}$ = 1710(50)(80) MeV.

The unquenched view of 0^{++} states

The old work by Weingarten and Lee (hep-lat/9910008) included a mixing matrix between the glue and $\overline{q}q$ states. In the quenched theory.

$$\begin{pmatrix} \mid f_0(1710)\rangle \\ \mid f_0(1500)\rangle \\ \mid f_0(1310)\rangle \end{pmatrix} = \begin{pmatrix} m_G & f & \sqrt{2}fr \\ f & m_S & 0 \\ \sqrt{2}fr & 0 & m_N \end{pmatrix} \begin{pmatrix} \mid G\rangle \\ \mid \overline{s}s\rangle \\ \frac{1}{\sqrt{2}}(\overline{u}u + \overline{d}d) \end{pmatrix}$$

Weingarten and Lee predicted that $f_0(1710)$ was 74(10) % 0++ glueball.

In unquenched QCD, both glue and $\overline{q}q$ states will couple to singlet 0^{++} hadrons, so better to do a variational fit with both types of operators as basis interpolating operators. Also need $\overline{qq}qq$ interopolating operators.

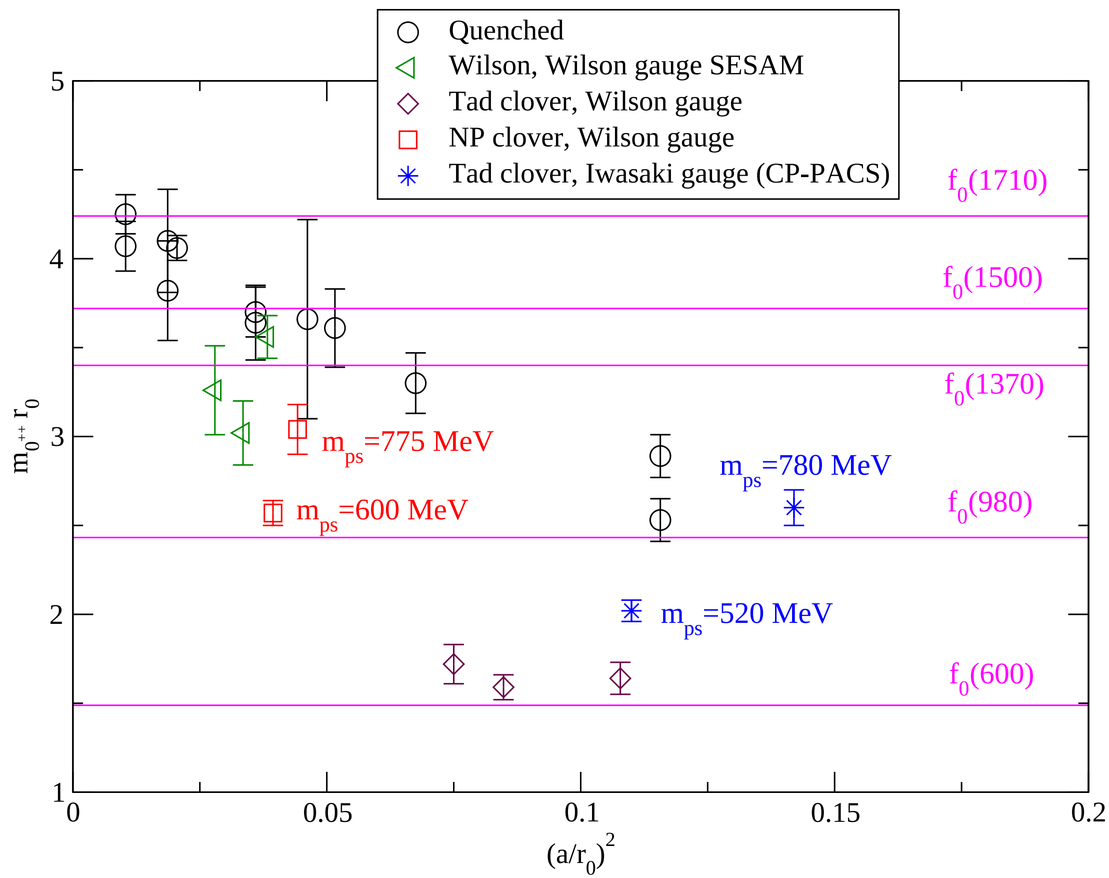
Two choices:

- Weak mixing, glueball mixes with $f_0(1710)$ and $f_0(1500)$
- strong mixing, perhaps quenched glueball contributes to $f_0(600)$ or $f_0(980)$

Flavour singlet 0^{++} masses

From Hart et al. hep-lat/0608026. Combined fit to 0^{++} glue and $\overline{q}q$ interpolating oprators.

Also configurations from CP-PACS with the Iwasaki gauge action



Attempts at the $f_0(600)$ (σ)

- Kentucky group, hep-ph/0607110. Used $\overline{\psi}\gamma_5\psi\overline{\psi}\gamma_5\psi$ interpolating operator in a quenched QCD calculation with pion masses as low as 180 MeV. Three state fit ($\pi(0)\pi(p=0)$, $f_0(600)$, $\pi(p=\frac{2\pi}{L})\pi(p=\frac{-2\pi}{L})$ with Bayes adaptive curve fitting algorithm. Their result $m_{f_0(600)} \sim 550$ MeV.

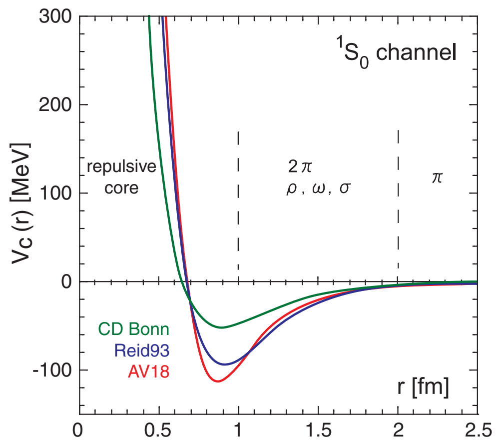
They fit to noisy data:

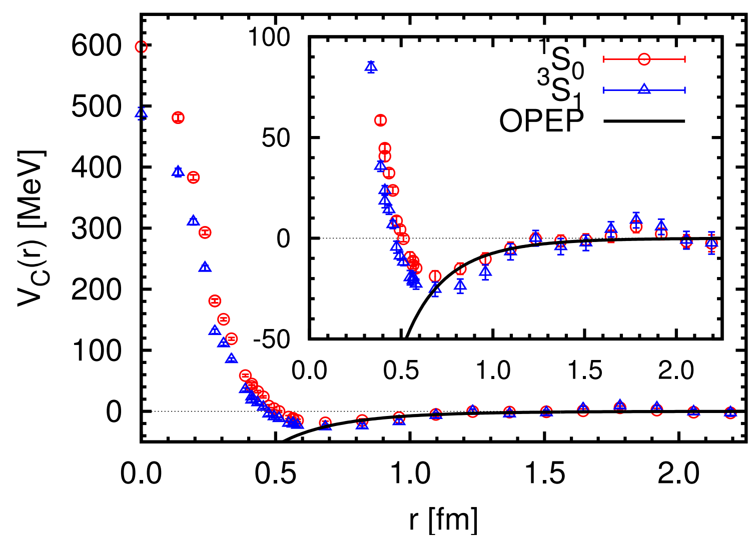
$$c(t) = a_0 e^{-m_0 t} + a_1 e^{-m_1 t} + a_2 e^{-m_2 t}$$

- The Bayes adaptive fitting method was used by Kentucky group to study the Roper resonance in quenched QCD. Other groups have not confirmed this (eg. Lasscock et al., arXiv:0705.0861).
- Can quenched QCD really be used to calculate the mass of a meson with a mass of 440 MeV and width of 544 MeV? Mathur et al. did get the mass of the $K^\star(1430)$ to be 1.41 ± 0.12. The width of $K^\star(1430)$ is 290 MeV.

Nuclear forces from lattice QCD

Ishii, Aoki, Hatsuda, Phys.Rev.Lett.99:022001,2007





Conclusions

- Many lattice collaborations now have access to full lattice QCD calculations with 300 MeV pions. This should help the study of exotic and flavour singlet mesons on the lattice.
- In lattice QCD we are starting to deal with resonances, such as the b_1 meson that have decayed in our lattice calculations.
- Need the value of mass of 1^{-+} from unquenched QCD, that includes the effects of the decay thresholds.
- Need spectroscopy of multi-quark 1^{-+} from unquenched QCD. Something like the calculation by the Kentucky lattice group of 0^{++} hep-ph/0607110, but with full QCD.
- Need to redo lattice calculations of the flavour singlet 0^{++} mesons with lighter quarks and more lattice spacings.

Given that prediction is cooler than postdiction (prediction after the fact)), the lattice calculations need to be improved before there are new results from Gluex and PANDA.